Logarithm

Take the exponential function $f(x) = b^x$. Suppose you are given a $y > 0$. Then how can you find the x such that $y = b^x$. In other words what is the inverse of the exponential function $f(x) = b^x$?

Def. The logarithm function with base b is $f(x) = \log_b x$ where $y = \log_b x$ if $x = b^y$.

$$y = \log_b x$$

The exponent you need to raise b to to get x is y.

Ex.

$\log_2 8 = ?$

Ans. $2^? = 8$

We know $2^3 = 8$.

so $\log_2 8 = 3$

$\log_9 3 = ?$

Ans. $9^? = 3$

We know $9^{1/2} = 3$.

$\therefore \log_9 3 = \frac{1}{2}$.

$\log_5\left(\frac{1}{25}\right) = ?$

Ans. $5^? = \frac{1}{25}$

We know $5^{-2} = 1/25$.

$\therefore \log_5\left(\frac{1}{25}\right) = -2$

Exercise

Find $\log_3 9$

$\log_{16} 4$

$\log_2\left(\frac{1}{8}\right)$

Example

Write in logarithm form.

(a) $16 = 2^4$

The exponent you need to raise 2 to get 16 is 4.

Thus, $\log_2 16 = 4$

(b) $9 = \sqrt{81}$

$\Rightarrow 9 = 81^{1/2}$

The exponent you need to raise 81 to get 9 is $\frac{1}{2}$.

Thus, $\log_{81} 9 = \frac{1}{2}$.

(c) $x^a = z$

The exponent you need to raise x to get z is a.

$\therefore \log_x z = a$.

Exercise :

$$\frac{1}{49} = 7^{-2}.$$

Examples

Find

(a) $\log_{169} 13$

Let $\log_{169} 13 = x.$

Then, $169^x = 13$

$(13^2)^x = 13$ $\qquad (13^2 = 169)$

$13^{2x} = 13$

Hence, $2x = 1$

$\Rightarrow x = \frac{1}{2}.$

$\therefore$, $\log_{169} 13 = \frac{1}{2}$

Exercise

Find $\log_{10} 1000$

$\log_5 \left(\frac{1}{5}\right)$

When the base b for the logarithm is e, we call it natural logarithm and $\log_e$ is written as $\ln$

Ex. Find

$\ln 1$

Ans. $e^0 = 1$. Thus, $\ln 1 = 0$

$\ln(-2)$.

Ans. Let $\ln(-2) = x$. Then

$e^x = -2$

But e^x is always positive. So this is undefined.

$\ln(415)$

$\ln(415) \approx 6.0283$

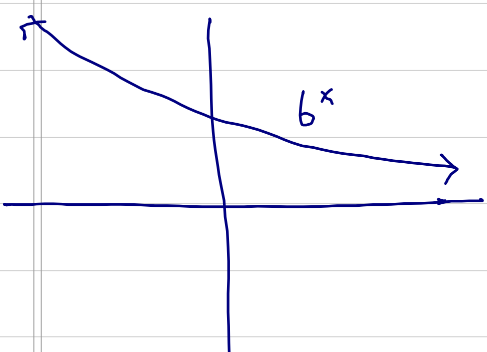
Graphs of Logarithm

Recall:

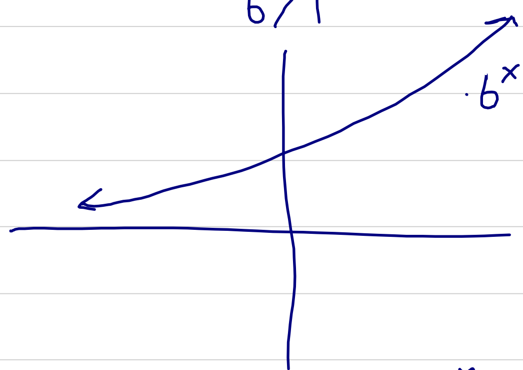
1) log is inverse of exp $f(x) = b^x$.
2) The graph of inverse is given by reflecting about the line $y = x$.

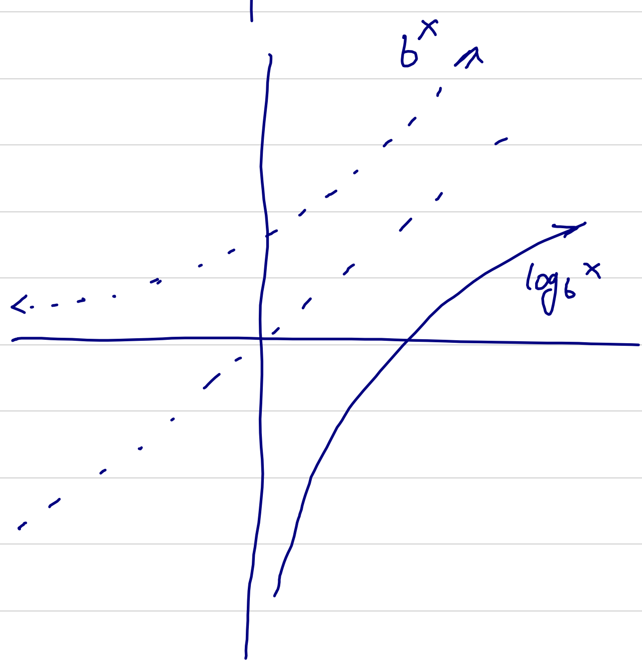
The exponential function $f(x) = b^x$ has essentially two types of graphs according to whether $0 < b < 1$ or $b > 1$.

$0 < b < 1$



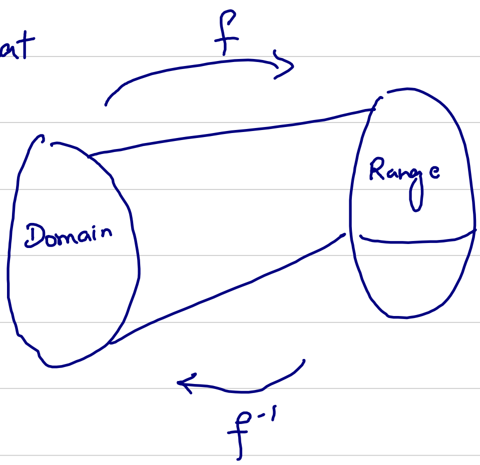
$b > 1$





We will skip the transformation of graphs of logarithms. You should be able to do this whenever you encounter such a situation in Calculus or later in life.

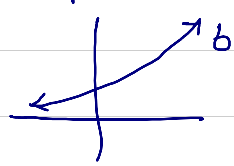
Recall that



Domain of f^{-1} = Range of f

Since $\log_b(x)$ is the inverse of b^x, and range of b^x is $(0, \infty)$, the

Domain of $\log_b(x)$ is $(0, \infty)$



Exercise

Find domain of

(a) $f(x) = \log_b(x-4)$

Soln. $x - 4 > 0$

$x > 4$

$\therefore$ Domain of $f(x) = (4, \infty)$

(b) $g(x) = \log_b(5-2x)$

Soln. $5 - 2x > 0$

$5 > 2x$

$5/2 > x$

$\therefore$ Domain of $f = (-\infty, 5/2)$

Exercise:

Find domain of the following logarithm functions:

(a) $f(x) = \log_b(x+2)$

(b) $g(x) = \log_b(3-5x)$